# White Paper: A Pragmatically-Modified Black-Litterman Approach to Help Multi-Asset Portfolios to Cope with Low-Probability "Black Swan" Scenarios

## *Overview:*

Portfolio manager John manages an institutional fund of $45 billion. The fund has no explicit liability profile to match and does not contain any investment with a near-term maturity date. It is a partially illiquid fund, but historically only a minor percentage of its holdings may need to be withdrawn based on the needs of its sponsors. Therefore, its investments cannot be wholly illiquid. The strategic allocation of the fund can be modeled by a U.S. multi-asset portfolio, which produces a desirable historical portfolio return of 7.79%, as shown in **Figure 1**. John's goal is to generate a steady absolute return over the medium to long term. The sponsor is well aware that markets may swing up and down in the short term, and that artificially creating an absolute return vehicle may be very costly and hurt returns without achieving the sponsor's longer-term investment objectives.

| | Description | | | Position | Weight | Market | Risk | Return |
|---|---|---|---|---|---|---|---|---|
| No. | Product | Asset Class | Currency | Actual | Actual | Last Price | Volatility | Historical |
| 1 | American Funds Growth Fund of Amer A | Equity | USD | 4,669,000,000.00 | 10.29% | 45.95 | 12.98% | 13.57% |
| 2 | DoubleLine Total Return Bond Fund | Fixed Income | USD | 2,196,000,000.00 | 4.84% | 11.02 | 2.53% | -0.31% |
| 3 | Fortress Investment Group LLC | Hedge Funds | USD | 2,214,000,000.00 | 4.88% | 7.07 | 32.31% | 27.78% |
| 4 | Franklin Templeton Hard Strategy A | Foreign Exchange | USD | 1,856,000,000.00 | 4.09% | 9.09 | 4.01% | -0.24% |
| 5 | Goldman Sachs Commodity Strategy A | Commodity | USD | 2,710,000,000.00 | 5.97% | 4.86 | 10.27% | -1.72% |
| 6 | PIMCO Total Return A | Fixed Income | USD | 3,285,000,000.00 | 7.24% | 10.94 | 2.57% | 1.85% |
| 7 | PowerShares Global Listed Private Eq | Private Equity | USD | 2,346,000,000.00 | 5.17% | 10.96 | 13.91% | 7.77% |
| 8 | SPDR Gold Shares | Commodity | USD | 1,086,570,000.00 | 2.39% | 118.1 | 12.01% | -1.27% |
| 9 | SPDR MSCI ACWI ex-US ETF | Equity | USD | 3,676,000,000.00 | 8.10% | 34.52 | 11.47% | 1.50% |
| 10 | SPDR Russell Small Cap Completeness | Equity | USD | 2,605,800,000.00 | 5.74% | 84.21 | 14.70% | 14.55% |
| 11 | SPDR S&P 500 ETF Trust | Equity | USD | 10,029,500,000.00 | 22.10% | 198.41 | 10.90% | 13.11% |
| 12 | T. Rowe Price US Treasury Long Term | Fixed Income | USD | 3,780,000,000.00 | 8.33% | 12.89 | 8.74% | 5.38% |
| 13 | iShares Core Total US Bond Market ETF | Fixed Income | USD | 3,059,280,000.00 | 6.74% | 110.14 | 2.62% | 2.40% |
| 14 | iShares Dow Jones US Real Estate (ETF) | Equity | USD | 1,867,250,000.00 | 4.11% | 74.34 | 10.09% | 4.65% |
| | | | | **45,380,400,000.00** | **100.00%** | | | **7.79%** |

Figure 1: US Multi-Asset Funds

## *Investment Problem:*

In order to reach his investment goal, John faces several significant limitations associated with the characteristics of any multi-asset portfolio. Firstly, it is common knowledge among professional investors that most institutional-sized portfolios derive their returns from beta instead of alpha [1] due to their massive size. Secondly, any significant change to the portfolio will result in high transaction costs in terms of market impact, which will offset the benefits from trying to achieve meaningful returns[2]. In addition, hedging[3] may not be a practical strategy for a large portfolio as

[1] Alpha is a risk-adjusted measure of active return on an investment. A positive alpha of 1.0 means the fund has outperformed its benchmark index by 1%. Correspondingly, a similar negative alpha would indicate an underperformance of 1%.
[2] An active return is the difference between the benchmark and the actual return. It can be positive or negative and is typically used to assess performance.
[3] Hedging is to make an investment to reduce the risk of adverse price movement in an asset.

such, because the massive size of any short positions required by the hedge may artificially induce a market crash. Finally, there is always some portion of investments in any portfolio that may be difficult to sell *en masse*, such as those in real estate investment trusts.

After the financial crisis in 2008, the sponsor of the fund introduced the mandate to account for potential "Black Swan" scenarios in its routine portfolio construction reviews, because it may be impossible to make any sudden, major changes to such a large portfolio or put on any hedge even with, say, 6 months of visibility in the event that such a "Black Swan" event does happen. At the same time, the fund cannot and will not generate a healthy medium to long-term return if it is "permanently" positioned to avoid extreme events. John wants to find an optimal asset allocation that can take into account low-probability extreme events for a multi-asset portfolio.

**John's "Black Swan" Scenarios:** On July 30th 2014, Argentina defaulted again by not paying its bondholders after its last default 12 years ago. (See **Appendix Sec I.1**) John is concerned that Argentina's default can trigger ripple effects in the global financial markets and may significantly impact his portfolio's performance. In order to protect the sponsor's wealth from excessive risk exposures and potential devaluation impacts, John wants to estimate what needs to be done to rebalance such a portfolio. Therefore, he comes up with three possible market outcomes from Argentina's default, which he describes as "Selective Default", "Managed Default", and "Destructive Default" scenarios with different shocks on chosen market indices (**Figure 2**).

**Argentina Selective Default**

Save Scenario | Delete Scenario | Description | Factor Significance

| Type | Index | Current | New | Change | Change % |
|---|---|---|---|---|---|
| Equity | iShares MSCI Emerging Markets | 44.16 | 42.84 | -1.32 | -3.00 |
| Bond | SPDR Barclays International Tre | 60.430 | 61.639 | 1.209 | 2.00 |

**Argentina Managed Default**

Save Scenario | Delete Scenario | Description | Factor Significance

| Type | Index | Current | New | Change | Change % |
|---|---|---|---|---|---|
| Equity | iShares MSCI Emerging Markets | 44.16 | 46.37 | 2.21 | 5.00 |
| Bond | SPDR Barclays International Tre | 60.430 | 58.617 | -1.813 | -3.00 |

**Argentina Destructive Default**

Save Scenario | Delete Scenario | Description | Factor Significance

| Type | Index | Current | New | Change | Change % |
|---|---|---|---|---|---|
| Equity | iShares MSCI Emerging Markets | 44.16 | 39.74 | -4.42 | -10.00 |
| Bond | SPDR Barclays International Tre | 60.430 | 56.804 | -3.626 | -6.00 |

Figure 2: Three Argentina default scenarios with market shocks

## *Practical Criteria*

John starts with the classic approach of maximizing portfolio Sharp Ratio. He first obtains the expected return of each asset under each scenario (See **Appendix Sec II 4**). Based on these expected returns, he calculates the optimal asset allocation under each scenario. The absolute changes in portfolio positions total from 139% to 166%, and the resulting portfolios concentrate on a few assets (See **Appendix Sec II 3 & 4**). The key disadvantages of this naïve

approach are: a) not knowing which scenario to choose in advance and b) the potentially high transaction costs if he changes his views. John is also concerned about several practical considerations before choosing a specific implementation:

1. If the portfolio is structured based on any specific black swan scenario, it will result in a "corner solution" concentrated in a few assets, rather than a full range of diversified assets. From one corner solution to the next (after a change in investment view) will result in yet another major change in weights; this approach may not be practical because no one knows exactly which scenario may happen.
2. The existing market views should be medium to long term in order to be consistent with investment goal. Otherwise, the high transaction costs from dramatic moves from existing scenarios will exceed any expected benefits from portfolio rebalancing.
3. There should be some evidence of appropriate trade-offs between risks and return.

## *Solution*

In order to address the practical criteria above, John is going to use the Black-Litterman model to obtain an optimal asset allocation. He picks the *Argentina Managed Default* scenario and defines four user inputs for this model (See **Appendix Section II 5**). By varying only the confidence level, John compares the different sets of optimal asset allocation and believes that the confidence level of 80-95%, resulting in a total weight change of a few percentages, may keep the overall weight adjustments to be relatively reasonable (See **Figure 3**).

| CL | 0% (Tau =0.01) | | 20% (Tau =0.01) | | 40% (Tau =0.01) | | 60% (Tau =0.01) | | 80% (Tau =0.01) | | 95% (Tau =0.01) | | 100% (Tau =0.01) | | 95% (Tau =0.1) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Asset | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg | Weight | Abs Chg |
| 1 | 10.29% | 0.00% | 10.29% | 0.00% | 10.27% | 0.02% | 10.26% | 0.03% | 10.20% | 0.09% | 9.93% | 0.36% | 3.38% | 6.91% | 8.15% | 2.14% |
| 2 | 4.84% | 0.00% | 4.84% | 0.00% | 4.85% | 0.01% | 4.86% | 0.02% | 4.90% | 0.06% | 5.10% | 0.26% | 0.00% | 4.84% | 4.05% | 0.79% |
| 3 | 4.88% | 0.00% | 4.86% | 0.02% | 4.83% | 0.05% | 4.77% | 0.11% | 4.58% | 0.30% | 3.58% | 1.30% | 0.00% | 4.88% | 0.00% | 4.88% |
| 4 | 4.09% | 0.00% | 4.09% | 0.00% | 4.10% | 0.01% | 4.11% | 0.02% | 4.15% | 0.06% | 4.33% | 0.24% | 9.95% | 5.86% | 6.07% | 1.98% |
| 5 | 5.97% | 0.00% | 5.97% | 0.00% | 5.98% | 0.01% | 5.99% | 0.02% | 6.01% | 0.04% | 6.14% | 0.17% | 6.50% | 0.53% | 6.20% | 0.23% |
| 6 | 7.24% | 0.00% | 7.25% | 0.01% | 7.26% | 0.02% | 7.28% | 0.04% | 7.36% | 0.12% | 7.74% | 0.50% | 7.16% | 0.08% | 6.70% | 0.54% |
| 7 | 5.17% | 0.00% | 5.16% | 0.01% | 5.15% | 0.02% | 5.13% | 0.04% | 5.07% | 0.10% | 4.73% | 0.44% | 0.00% | 5.17% | 0.42% | 4.75% |
| 8 | 2.39% | 0.00% | 2.40% | 0.01% | 2.41% | 0.02% | 2.42% | 0.03% | 2.48% | 0.09% | 2.77% | 0.38% | 7.37% | 4.98% | 4.40% | 2.01% |
| 9 | 8.10% | 0.00% | 8.09% | 0.01% | 8.09% | 0.01% | 8.07% | 0.03% | 8.02% | 0.08% | 7.76% | 0.34% | 6.47% | 1.63% | 7.54% | 0.56% |
| 10 | 5.74% | 0.00% | 5.73% | 0.01% | 5.73% | 0.01% | 5.71% | 0.03% | 5.66% | 0.08% | 5.39% | 0.35% | 0.00% | 5.74% | 3.39% | 2.35% |
| 11 | 22.10% | 0.00% | 22.10% | 0.00% | 22.07% | 0.03% | 22.05% | 0.05% | 21.96% | 0.14% | 21.53% | 0.57% | 13.45% | 8.65% | 18.25% | 3.85% |
| 12 | 8.33% | 0.00% | 8.35% | 0.02% | 8.38% | 0.05% | 8.44% | 0.11% | 8.62% | 0.29% | 9.58% | 1.25% | 25.22% | 16.89% | 13.23% | 4.90% |
| 13 | 6.74% | 0.00% | 6.75% | 0.01% | 6.76% | 0.02% | 6.79% | 0.05% | 6.86% | 0.12% | 7.28% | 0.54% | 14.83% | 8.09% | 17.25% | 10.51% |
| 14 | 4.12% | 0.00% | 4.12% | 0.00% | 4.12% | 0.00% | 4.12% | 0.00% | 4.13% | 0.01% | 4.14% | 0.02% | 5.66% | 1.54% | 4.36% | 0.24% |
| Total Portfolio Change | 100.00% | 0.00% | 100.00% | 0.10% | 100.00% | 0.28% | 100.00% | **0.58%** | 100.00% | 1.58% | 100.00% | 6.72% | 100.00% | 75.80% | 100.00% | 39.73% |

Figure 3: Optimal Asset Weight Allocation by Black-Litterman Model with Different Confidence Level (tau = 0.01)
Note: Last column is calculated with tau =0.1 and confidence level 95%

### Multiple Scenarios

Even though John can use the Black-Litterman model to get the desired outcome for one particular scenario, the textbook approach may not give him a pragmatic solution. He needs some flexibility in order to combine multiple market views, including the low probability "Black Swan" scenarios. Therefore, John modified the Black-Litterman model slightly to produce a single set of expected returns for the portfolio under the combined views, which is a more stable and accurate asset allocation after applying reversed optimization[4]. As before, he chooses a confidence level of 90% and a Tau of 0.1.

| Asset | New Weights | Original Weights | Abs Chg |
|---|---|---|---|
| 1 | 2.04% | 10.29% | 8.3% |
| 2 | 3.27% | 4.84% | 1.6% |
| 3 | 42.08% | 4.88% | 37.2% |
| 4 | 1.15% | 4.09% | 2.9% |
| 5 | 2.84% | 5.97% | 3.1% |
| 6 | 4.17% | 7.24% | 3.1% |
| 7 | 3.79% | 5.17% | 1.4% |
| 8 | 1.32% | 2.39% | 1.1% |
| 9 | 1.65% | 8.10% | 6.5% |
| 10 | 1.51% | 5.74% | 4.2% |
| 11 | 4.17% | 22.10% | 17.9% |
| 12 | 7.75% | 8.33% | 0.6% |
| 13 | 2.49% | 6.74% | 4.3% |
| 14 | 21.77% | 4.12% | 17.7% |
| Total Changes | | | 109.70% |

Figure 4: Optimal Weight Allocation under the Combined View by Black-Litterman Model

Next, John incorporates the two extreme views into the mainstream view: "Selective Default" view with a confidence level of 10% and "Destructive default" with a confidence level of 5%. His results are displayed in **Figure 4**. John then performs a simple "out of sample" test on the new portfolio. His original portfolio (as shown in **Figure 1**) yields a 5.33% upside under the *US Economic Recovery* scenario and a -29.54% downside under *Financial Crisis 2008* (See **Appendix Section II 6,** as shown in **Figure 6a**); the new portfolio yields 11.13% upside under the *US Economic Recovery* scenario and -32.86% downside under *Financial Crisis 2008* (as shown in **Figure 6b**). This is significant. In his original portfolio, a bad year will take almost 6 good years to "catch up"; in the improved portfolio, a bad year will take about 3 years to "catch up". The former will be considered a catastrophic loss by most sponsors, while the latter event is probably still considered a recoverable event. John is still concerned about the potentially significant changes in a few assets, but at least he is starting from a well-diversified portfolio of all 14 assets.

By combining his main view with 2 low-probability extreme events, his views are now more interesting than a single event view, so that it can be held for a longer horizon, allowing John to stick to his investment thesis and to take the extra time to make portfolio adjustments.

## *Conclusions*

John is happy to see that he can incorporate low-probability extreme events to construct a more stable portfolio. By incorporating this recommendation, his portfolio went from facing a catastrophic loss to a more recoverable one in the event of another "Black Swan". This more stable combined view can allow John to stick to his medium-to-long term investment thesis. With the help from a user-friendly platform, he can now afford to test out his investment ideas under different views and get them right prior to implementation.

[4] See **Appendix Sec II 3** for Reversed Optimization.

# Technical Appendices

## I. Definitions

1. Historical Background on Scenarios Used
   In 2001, Argentina had the largest sovereign debt default in history at $93 billion. Between 2005 and 2010, the government restructured its debts and agreed on a settlement with its bondholders to pay back 33 cents on the dollar. However, $4 billion remained unsettled, and some "vulture funds" had been holding out for Argentina to pay back their debts in full.

2. Three Scenarios about Argentina's Default

| | **Selective Default** | **Managed Default** | **Destructive Default** |
|---|---|---|---|
| **Action** | Make a better offer to vulture fund before the expiration of RUFO[*] | Wait until everyone accepts the 33 cents offer after the expiration of RUFO | Never reach an settlement |
| **Duration** | Default ends before Dec 31st 2014 | Default ends after Dec 31st 2014 | Indefinite |
| **Benefit** | Restore reputation in bond market | Conserve its reserves and recovery in the long run | No payment need to be made |
| **Risk** | May result $29 billion tab, which depletes almost its entire foreigner currency reserve | Damage its reputation in bond market in the short run | Unable to get access to capital market and further economic instability |
| **Probability** | Low (0% - 30%) | Medium (30% - 70%) | Low (0% - 30%) |
| **Overall Assessment** | Negative Scenario | Positive Scenario | Worst Scenario |

*Note: RUFO is short for 'Rights upon Future Offer' Clause on Argentina's bond contract, which gives right to certain bondholders to enjoy the same benefits as the other bondholders as well as those 'vulture funds'.

3. The Three Views under Each Scenario are defined as:

| | Market Factors Expressing this View | View 1 – Selective Default | View 2 – Managed Default | View 3 – Destructive Default |
|---|---|---|---|---|
| 1. Asset | iShare MSCI Emerging Market | -3% | +5% | -10% |
| 2. Equity | SPDR Barclay International Treasury | +2% | -3% | -6% |
| Confidence Level | N/A | 10% | 90% | 5% |

## II. Formulae

Mathematical Model and Results

1. Basic Modelling Parameters
   a. This case study uses 4 years of historical data from May 2010 to May 2014
   b. In Figure 1, the annual historical return for each asset is calculated using the forward 6-month rolling windows over the 4 years used. For consistency, all the statistics are computed over the same horizon, but they may be annualized as and when appropriate.
   c. The risk-free rate of return is set to be 0.1% during this period, and is assumed to be constant.
2. Sharp Ratio
   $Sharp\ Ratio = \frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$, where Ra is the asset return, Rrf is the risk-free asset return and risk of the portfolio is measured by the variance of excessive return. The Sharp Ratio tells an investor whether a higher return of a portfolio is achieved via better investment decisions or from taking excessive risk. The greater a portfolio's Sharp Ratio is, the better its risk-adjusted performance has been.

3. Portfolio Optimization and Reversed Optimization
   In this case study, portfolio optimization is used to find the optimal asset weight allocation that maximizes/minimizes an objective function under certain constraints (e.g. weights cannot be negative), given the expected return of each asset under each scenario. Reversed portfolio optimization calculates the implied return of each asset in the portfolio, so that the current weights are considered optimal under a specific choice of objective function.

4. Naïve Modelling
   In this case study, our Naïve Modelling strategy is to optimize the portfolio by maximizing portfolio Sharp Ratio under each scenario. The model is implemented in two steps:
   1. First Step: Calculate Expected Returns under Each View
   The expected return for each asset under each view is calculated by individually running a regression on each asset's return against the market factor returns under each view as shown in Figure 2 and Appendix Sec I.3. The result is displayed in the "Asst Chg" column in Figure 6.
   2. Second Step: Calculate the optimal weight allocation
   Solve for $\mathbf{w}= [w_1, w_2,\cdots w_{14}]^T$, in order to maximize the objective function $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$, subject to:
   Constraint 1: $w_1+w_2 \cdots+w_{14} = 1$
   Constraint 2: $w_i \geq 0$ for i=1, 2,⋯ 14

   Results from naïve modeling:

| Scenario | Original | Selective Default | Managed Default | Destructive Default |
|---|---|---|---|---|
| **$\mu$-portfolio return** | 10.16% | 2.12% | 3.78% | 3.72% |
| **$\sigma$-portfolio Std** | 10.11% | 6.21% | 8.39% | 6.95% |
| **Sharp Ratio** | 1.00 | 0.34 | 0.45 | 0.54 |

Note: The precise calculation procedures and their results are available upon request.

| | | Original | Selective Default | | Managed Default | | Destructive Default | |
|---|---|---|---|---|---|---|---|---|
| **No.** | Asset Name | %Weight | %Weight | % Abs Change | %Weight | % Abs Change | %Weight | % Abs Change |
| **1** | AGTHX-American Funds Growth Fund of Amer A | 10.29% | - | 10.29% | - | 10.29% | - | 10.29% |
| **2** | DLTNX-DoubleLine Total Return Bond Fund | 4.84% | - | 4.84% | **65.31%** | 60.47% | - | 4.84% |
| **3** | FIG-Fortress Investment Group LLC | 4.88% | - | 4.88% | **13.03%** | 8.15% | - | 4.88% |
| **4** | ICPHX-Franklin Templeton Hard Currency FUND | 4.09% | **5.08%** | 0.99% | - | 4.09% | - | 4.09% |
| **5** | GSCAX-Goldman Sachs Commodity Strategy A | 5.97% | **3.10%** | 2.87% | - | 5.97% | - | 5.97% |
| **6** | PTTAX-PIMCO Total Return A | 7.24% | - | 7.24% | - | 7.24% | - | 7.24% |
| **7** | PSP-PowerShares Global Listed Private Eq | 5.17% | - | 5.17% | **1.57%** | 3.60% | - | 5.17% |
| **8** | GLD-SPDR Gold Shares | 2.39% | **15.63%** | 13.24% | - | 2.39% | - | 2.39% |
| **9** | CWI-SPDR MSCI ACWI ex-US ETF | 8.10% | - | 8.10% | - | 8.10% | - | 8.10% |
| **10** | RSCO-SPDR Russell Small Cap Completeness | 5.74% | - | 5.74% | **20.08%** | 14.34% | - | 5.74% |
| **11** | SPY-SPDR S&P 500 ETF Trust | 22.10% | - | 22.10% | - | 22.10% | **26.05%** | 3.95% |
| **12** | PRULX-T. Rowe Price US Treasury Long-Term | 8.33% | **33.89%** | 25.56% | - | 8.33% | **73.95%** | 65.62% |
| **13** | AGG-iShares Core Total US Bond Market ETF | 6.74% | **42.31%** | 35.57% | - | 6.74% | - | 6.74% |
| **14** | IRY-iShares Dow Jones US Real Estate (ETF) | 4.12% | - | 4.12% | - | 4.12% | - | 4.12% |
| | % Portfolio 1 Asset weight absolute Change | | | **150.70%** | | **165.93%** | | **139.14%** |

Figure 5: Optimal Weight Allocations by Naïve Maximization of Sharp Ratio under the Three Views

Buy | Sell | Fields: Scenarios | Calc. Horizon: 5 Year | Use Active Return | Include Cash | Use Weekly | Import | Export

| Description | | Position | Weight | Return | Argentina Managed Default | | | Argentina Selective Default | | | Argentina Destructive Default | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Product | Asset Class | Actual | Actual | Historical... | Pos Chg | Asst Chg | Scen Prc | Pos Chg | Asst Chg | Scen Prc | Pos Chg | Asst Chg | Scen Prc |
| American Funds Growth Fund of Am... | Equity | 4,669,643,16... | 10.29% | 13.57% | 0.51% | 4.99% | 48.24 | -0.32% | -3.09% | 44.53 | -0.41% | -3.99% | 44.11 |
| DoubleLine Total Return Bond Fund | Fixed Inco... | 2,196,411,36... | 4.84% | -0.31% | 0.00% | 0.01% | 11.02 | 0.00% | 0.02% | 11.02 | -0.06% | -1.31% | 10.88 |
| Fortress Investment Group LLC | Hedge Fun... | 2,214,563,52... | 4.88% | 27.78% | 0.86% | 17.62% | 8.32 | -0.54% | -11.02% | 6.29 | -0.41% | -8.50% | 6.47 |
| Franklin Templeton Hard Currency F... | Foreign Ex... | 1,856,058,36... | 4.09% | -0.24% | -0.01% | -0.19% | 9.07 | 0.01% | 0.18% | 9.11 | -0.16% | -3.93% | 8.73 |
| Goldman Sachs Commodity Strategy A | Commodity | 2,709,209,88... | 5.97% | -1.72% | 0.09% | 1.45% | 4.93 | -0.05% | -0.84% | 4.82 | -0.29% | -4.80% | 4.63 |
| PIMCO Total Return A | Fixed Inco... | 3,285,540,96... | 7.24% | 1.85% | -0.06% | -0.79% | 10.85 | 0.04% | 0.52% | 11.00 | -0.08% | -1.16% | 10.81 |
| PowerShares Global Listed Private Eq | Private Eq... | 2,346,166,68... | 5.17% | 7.77% | 0.40% | 7.78% | 11.81 | -0.25% | -4.76% | 10.44 | -0.53% | -10.26% | 9.84 |
| SPDR Gold Shares | Commodity | 1,084,591,56... | 2.39% | -1.27% | -0.14% | -5.76% | 111.29 | 0.09% | 3.71% | 122.48 | -0.09% | -3.57% | 113.88 |
| SPDR MSCI ACWI ex-US ETF | Equity | 3,675,812,40... | 8.10% | 1.50% | 0.44% | 5.40% | 36.38 | -0.27% | -3.28% | 33.39 | -0.67% | -8.28% | 31.66 |
| SPDR Russell Small Cap Completeness | Equity | 2,604,834,96... | 5.74% | 14.55% | 0.36% | 6.35% | 89.56 | -0.23% | -3.94% | 80.89 | -0.28% | -4.86% | 80.12 |
| SPDR S&P 500 ETF Trust | Equity | 10,029,068,4... | 22.10% | 13.11% | 0.98% | 4.42% | 207.18 | -0.61% | -2.76% | 192.94 | -0.54% | -2.46% | 193.52 |
| T. Rowe Price US Treasury Long-Term | Fixed Inco... | 3,780,187,32... | 8.33% | 5.38% | -0.45% | -5.38% | 12.20 | 0.28% | 3.31% | 13.32 | 0.50% | 5.99% | 13.66 |
| iShares Core Total US Bond Market ... | Fixed Inco... | 3,058,638,96... | 6.74% | 2.40% | -0.09% | -1.40% | 108.60 | 0.06% | 0.88% | 111.11 | 0.03% | 0.46% | 110.65 |
| iShares Dow Jones US Real Estate (... | Equity | 1,869,672,48... | 4.12% | 4.65% | 0.11% | 2.69% | 76.34 | -0.07% | -1.64% | 73.12 | -0.16% | -3.77% | 71.54 |
| Cash in USD | Cash | 0.00 | 0.00% | 0.06% | 0.00% | 0.02% | 1.00 | 0.00% | -0.01% | 1.00 | 0.00% | 0.02% | 1.00 |
| **Total** | | 45,380,400,0... | 100.00% | 7.79% | 3.01% | | | -1.85% | | | -3.15% | | |

Figure 6: Shocks to the Original Portfolio under the Three Argentina Default Scenarios

5. Enhanced Modelling Approach – Modified Black-Litterman
    a. Key assumption: Current portfolio weight (wMkt) is a sufficiently good estimator to the equilibrium market weight.

b. User Inputs:

| | |
|---|---|
| **P** | Each column of matrix P represents an investor's view about the assets' weighted expected returns, which all the elements of the column sum up to 1 |
| **Q** | A column vector, in which an element represents a total change of a portfolio's weighted expected returns under a particular view |
| **Confidence Level** | A column vector, in which an element represents the investor's confidence in the corresponding view |
| **Tau** | A scalar for an investor to control the maximal change of the portfolio's by looking at the absolute portfolio weight change at 95% confidence level produced by Black-Litterman model |

c. The model is executed on Matlab in four steps.
   i. Compute P Matrix
   Each column of P matrix represents a view and is calculated independently in the same process. First, "Pos Chg" column in Figure 3 is produced by the weighted "Asst Chg". Then, elements in the column is scaled to sum up to 100%.
   ii. Compute Implied Return Π
   Given wMkt of the current portfolio, use reversed optimization to find the vector of implied returns Π, such that the returns maximize $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$. The close form solution for the reversed optimization is $\Pi = \frac{\Sigma * wMkt * (\mu^T * wMkt)}{wMkt^T * \Sigma * wMkt}$, where $\Sigma$ is the covariance matrix of asset returns and $\mu$ is asset historical returns.
   iii. Calculate the combined returns E[R] based on implied returns and the forward-looking views $\mathbb{E}(R) = \Pi + \tau\Sigma P^T[\boldsymbol{\Omega} + \tau P\Sigma P^T]^{-1}(Q - P\Pi)$, where $\boldsymbol{\Omega}$ is the diagonal covariance matrix of errors terms from the expressed views expressing the uncertainty in each view.
   iv. Given E[R], get the optimal asset weight allocation that maximizes $\frac{E[Ra-Rrf]}{\sqrt{Var[Ra-Rrf]}}$. We solve the problem numerically to account for non-negative positions, but the results should be similar to the following closed-form solution:
   $\mathbf{w}_{\text{optimal}} = \frac{\Sigma^{-1} * E[R]}{I^T * \Sigma^{-1} * E[R]}$, where I is an identical matrix that has the same size as E[R]

d. Mathematical Properties of the Model
   i. When the confidence level is 0% for every view, the optimal weight is equal to wMkt and the E[R] =Π. When confidence level is 0% in the setting of the single managed default view, the portfolio return is 10.16% which is consistent with the original portfolio return in Figure 1. When there are no additional views added, the model gives the original asset weight allocation. Please find our results below:

| No. | Optimal Weight | Implied Return = Π |
|---|---|---|
| **1** | 10.29% | 13.59% |
| **2** | 4.84% | -0.29% |
| **3** | 4.88% | 34.02% |
| **4** | 4.09% | 5.51% |
| **5** | 5.97% | 11.59% |
| **6** | 7.24% | 1.22% |
| **7** | 5.17% | 19.55% |
| **8** | 2.39% | 2.09% |
| **9** | 8.10% | 15.94% |
| **10** | 5.74% | 15.83% |
| **11** | 22.10% | 12.93% |
| **12** | 8.33% | -6.36% |
| **13** | 6.74% | -0.52% |
| **14** | 4.11% | 13.59% |
| | **Portfolio Return:** | **10.16%** |

ii. The model always converges to a specific weight allocation when the confidence level is 100% and thus picking a good estimator of confidence level is important. The weight allocation is shown below

| Optimal Weight | Abs Chg. |
|---|---|
| 3.38% | 6.91% |
| 0.00% | 4.84% |
| 0.00% | 4.88% |
| 9.95% | 5.86% |
| 6.50% | 0.53% |
| 7.16% | 0.08% |
| 0.00% | 5.17% |
| 7.37% | 4.98% |
| 6.47% | 1.63% |
| 0.00% | 5.74% |
| 13.45% | 8.65% |
| 25.22% | 16.89% |
| 14.83% | 8.09% |
| 5.66% | 1.54% |
| 100.00% | 75.80% |

e. Additional Parameters about Three Cases used in this White Paper

1. Cases 1a and 1b : Single View- Managed Default

a. Inputs

| | |
|---|---|
| **Q** | **3.06%** |
| **Confidence Level** | **90%** |
| **Tau (Case 1a)** | **0.01** |
| **Tau (Case 1b)** | **0.1** |

| No. | Assets | P-Managed |
|---|---|---|
| 1 | AGTHX-American Funds Growth Fund of Amer A | **17.15%** |
| 2 | DLTNX-DoubleLine Total Return Bond Fund | **-0.02%** |
| 3 | FIG-Fortress Investment Group LLC | **28.87%** |
| 4 | ICPHX-Franklin Templeton Hard Currency FUND | **-0.35%** |
| 5 | GSCAX-Goldman Sachs Commodity Strategy A | **2.75%** |
| 6 | PTTAX-PIMCO Total Return A | **-1.99%** |
| 7 | PSP-PowerShares Global Listed Private Eq | **13.32%** |
| 8 | GLD-SPDR Gold Shares | **-4.72%** |
| 9 | CWI-SPDR MSCI ACWI ex-US ETF | **14.40%** |
| 10 | RSCO-SPDR Russell Small Cap Completeness | **12.18%** |
| 11 | SPY-SPDR S&P 500 ETF Trust | **32.80%** |
| 12 | PRULX-T. Rowe Price US Treasury Long-Term | **-14.89%** |
| 13 | AGG-iShares Core Total US Bond Market ETF | **-3.17%** |
| 14 | IRY-iShares Dow Jones US Real Estate (ETF) | **3.66%** |
| | Total | 100.00% |

b. Sharp Ratio Chart for Different Confidence Level

| Confidence Level | 0% (Tau=0.01) | 20% (Tau=0.01) | 40% (Tau=0.01) | 60% (Tau=0.01) | 80% (Tau=0.01) | 95% (Tau=0.01) | 100% (Tau=0.01) | 95% (Tau=0.1) |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 10.16% | 10.13% | 10.08% | 9.98% | 9.68% | 8.22% | 1.71% | 3.35% |
| $\sigma$ | 10.11% | 10.10% | 10.08% | 10.05% | 9.93% | 9.34% | 5.29% | 6.46% |
| **Sharp Ratio** | 1.00 | 1.0030 | 0.9997 | 0.9933 | 0.9745 | 0.8800 | 0.3229 | 0.5182 |

2. Case II Combined Views – a. Inputs

| No. | Assets | P-Selective Default | P-Managed Default | P-Destructive Default |
|---|---|---|---|---|
| 1 | AGTHX-American Funds Growth Fund of Amer A | **17.15%** | **17.25%** | **12.75%** |
| 2 | DLTNX-DoubleLine Total Return Bond Fund | **-0.02%** | **-0.08%** | **2.14%** |
| 3 | FIG-Fortress Investment Group LLC | **28.87%** | **29.35%** | **12.06%** |
| 4 | ICPHX-Franklin Templeton Hard Currency FUND | **-0.35%** | **-0.52%** | **5.48%** |
| 5 | GSCAX-Goldman Sachs Commodity Strategy A | **2.75%** | **2.56%** | **9.53%** |
| 6 | PTTAX-PIMCO Total Return A | **-1.99%** | **-2.15%** | **2.99%** |
| 7 | PSP-PowerShares Global Listed Private Eq | **13.32%** | **13.22%** | **17.06%** |
| 8 | GLD-SPDR Gold Shares | **-4.72%** | **-4.93%** | **3.23%** |
| 9 | CWI-SPDR MSCI ACWI ex-US ETF | **14.40%** | **14.22%** | **21.74%** |
| 10 | RSCO-SPDR Russell Small Cap Completeness | **12.18%** | **12.30%** | **8.60%** |
| 11 | SPY-SPDR S&P 500 ETF Trust | **32.80%** | **33.29%** | **16.15%** |
| 12 | PRULX-T. Rowe Price US Treasury Long-Term | **-14.89%** | **-14.89%** | **-15.89%** |
| 13 | AGG-iShares Core Total US Bond Market ETF | **-3.17%** | **-3.25%** | **-0.84%** |
| 14 | IRY-iShares Dow Jones US Real Estate (ETF) | **3.66%** | **3.63%** | **5.01%** |
| | Total | 100.00% | 100.00% | 100.00% |
| | Q | **-1.89%** | **3.06%** | **-3.05%** |
| | Confidence Level | **10%** | **90%** | **5%** |
| | Tau | **0.01** | | |

| | |
|---|---|
| **Portfolio Return** | 9.95% |
| **Portfolio Standard Deviation** | 10.03% |
| **Sharp Ratio** | 0.99 |

6. Shocks for Original & New portfolio under *U.S. Economy Recovery* scenario and *Financial Crisis 2008* Scenario

| Description | | Position | Weight | Return | US Economic Recovery | | | Financial Crisis 2008 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Product | Asset Class | Actual | Actual | Historical... | Pos Chg | Asset Chg | Scen Prc | Pos Chg | Asset Chg | Scen Prc |
| American Funds Growth Fund of Am... | Equity | 4,669,643,16... | 10.29% | 1.93% | 0.85% | 8.27% | 49.75 | -4.03% | -39.14% | 27.96 |
| DoubleLine Total Return Bond Fund | Fixed Inco... | 2,196,411,36... | 4.84% | 1.09% | 0.01% | 0.26% | 11.05 | -0.27% | -5.50% | 10.41 |
| Fortress Investment Group LLC | Hedge Fun... | 2,214,563,52... | 4.88% | -8.84% | 1.00% | 20.45% | 8.52 | -1.78% | -36.53% | 4.49 |
| Franklin Templeton Hard Currency F... | Foreign Ex... | 1,856,058,36... | 4.09% | -2.88% | 0.02% | 0.55% | 9.14 | -0.45% | -10.97% | 8.09 |
| Goldman Sachs Commodity Strategy A | Commodity | 2,709,209,88... | 5.97% | -13.68% | 0.02% | 0.26% | 4.87 | -2.56% | -42.95% | 2.77 |
| PIMCO Total Return A | Fixed Inco... | 3,285,540,96... | 7.24% | 1.03% | 0.07% | 1.01% | 11.05 | -1.30% | -17.92% | 8.98 |
| PowerShares Global Listed Private Eq | Private Eq... | 2,346,166,68... | 5.17% | -10.90% | 0.49% | 9.38% | 11.99 | -1.86% | -35.97% | 7.02 |
| SPDR Gold Shares | Commodity | 1,084,591,56... | 2.39% | -1.56% | -0.08% | -3.22% | 114.30 | -0.58% | -24.37% | 89.32 |
| SPDR MSCI ACWI ex-US ETF | Equity | 3,675,812,40... | 8.10% | -4.02% | 0.37% | 4.55% | 36.09 | -1.56% | -19.26% | 27.87 |
| SPDR Russell Small Cap Completeness | Equity | 2,604,834,96... | 5.74% | -4.18% | 0.60% | 10.43% | 92.99 | -2.81% | -48.87% | 43.05 |
| SPDR S&P 500 ETF Trust | Equity | 10,029,068,4... | 22.10% | 9.08% | 1.76% | 7.97% | 214.21 | -8.84% | -40.02% | 119.00 |
| T. Rowe Price US Treasury Long-Term | Fixed Inco... | 3,780,187,32... | 8.33% | 12.64% | -0.08% | -0.95% | 12.77 | -1.21% | -14.57% | 11.01 |
| iShares Core Total US Bond Market ... | Fixed Inco... | 3,058,638,96... | 6.74% | 3.68% | 0.02% | 0.32% | 110.49 | -0.66% | -9.74% | 99.42 |
| iShares Dow Jones US Real Estate (... | Equity | 1,869,672,48... | 4.12% | 16.12% | 0.28% | 6.79% | 79.39 | -1.63% | -39.65% | 44.87 |
| Cash in USD | Cash | 0.00 | 0.00% | 0.00% | 0.00% | 0.03% | 1.00 | 0.00% | 0.00% | 1.00 |
| **Total** | | 45,380,400,0... | 100.00% | 1.77% | 5.33% | | | -29.54% | | |

Figure 6a: Shocks of Original Portfolio:
*US Economy Recovery:* 5.33%

*Financial Crisis 2008:* -29.54%

| Description | | Position | Weight | Return | US Economic Recovery | | | Financial Crisis 2008 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Product | Asset Class | Actual | Actual | Historical... | Pos Chg | Asset Chg | Scen Prc | Pos Chg | Asset Chg | Scen Prc |
| American Funds Growth Fund of Am... | Equity | 925,750,160.00 | 2.04% | 1.93% | 0.17% | 8.27% | 49.75 | -0.80% | -39.14% | 27.96 |
| DoubleLine Total Return Bond Fund | Fixed Inco... | 1,483,939,08... | 3.27% | 1.09% | 0.01% | 0.26% | 11.05 | -0.18% | -5.50% | 10.41 |
| Fortress Investment Group LLC | Hedge Fun... | 19,096,072,3... | 42.08% | -8.84% | 8.60% | 20.45% | 8.52 | -15.37% | -36.53% | 4.49 |
| Franklin Templeton Hard Currency F... | Foreign Ex... | $21,874,600.00 | 1.15% | -2.88% | 0.01% | 0.55% | 9.14 | -0.13% | -10.97% | 8.09 |
| Goldman Sachs Commodity Strategy A | Commodity | 1,288,803,36... | 2.84% | -13.68% | 0.01% | 0.26% | 4.87 | -1.22% | -42.95% | 2.77 |
| PIMCO Total Return A | Fixed Inco... | 1,892,362,68... | 4.17% | 1.03% | 0.04% | 1.01% | 11.05 | -0.75% | -17.92% | 8.98 |
| PowerShares Global Listed Private Eq | Private Eq... | 1,719,917,16... | 3.79% | -10.90% | 0.36% | 9.38% | 11.99 | -1.36% | -35.97% | 7.02 |
| SPDR Gold Shares | Commodity | 599,021,280.00 | 1.32% | -1.56% | -0.04% | -3.22% | 114.30 | -0.32% | -24.37% | 89.32 |
| SPDR MSCI ACWI ex-US ETF | Equity | 748,776,600.00 | 1.65% | -4.02% | 0.08% | 4.55% | 36.09 | -0.32% | -19.26% | 27.87 |
| SPDR Russell Small Cap Completeness | Equity | 685,244,040.00 | 1.51% | -4.18% | 0.16% | 10.43% | 92.99 | -0.74% | -48.87% | 43.05 |
| SPDR S&P 500 ETF Trust | Equity | 1,892,362,68... | 4.17% | 9.08% | 0.33% | 7.97% | 214.21 | -1.67% | -40.02% | 119.00 |
| T. Rowe Price US Treasury Long-Term | Fixed Inco... | 3,516,981,00... | 7.75% | 12.64% | -0.07% | -0.95% | 12.77 | -1.13% | -14.57% | 11.01 |
| iShares Core Total US Bond Market ... | Fixed Inco... | 1,129,971,96... | 2.49% | 3.68% | 0.01% | 0.32% | 110.49 | -0.24% | -9.74% | 99.42 |
| iShares Dow Jones US Real Estate (... | Equity | 9,879,313,08... | 21.77% | 16.12% | 1.48% | 6.79% | 79.39 | -8.63% | -39.65% | 44.87 |
| Cash in USD | Cash | 0.00 | 0.00% | 0.00% | 0.00% | 0.03% | 1.00 | 0.00% | 0.00% | 1.00 |
| **Total** | | 45,380,400,0... | 100.00% | 0.37% | 11.13% | | | -32.86% | | |

Figure 6b: Shocks of New Portfolio:
*US Economy Recovery:* 11.13%

*Financial Crisis 2008:* -32.86%

7. Other additional objective functions that are supported by the HedgeSPA platform includes:
   - Minimization of Value at Risk
   - Minimization of Controlled Value at Risk at 95% percentile
   - Minimization of Portfolio Variance
   - Minimization of Maximum Drawdown
   - Maximization of Alternative Sharp Ratio

   These objective functions are able to attain a better measure of portfolio upside against its downside, thus potentially providing an even more powerful solution for users to incorporate extreme views.

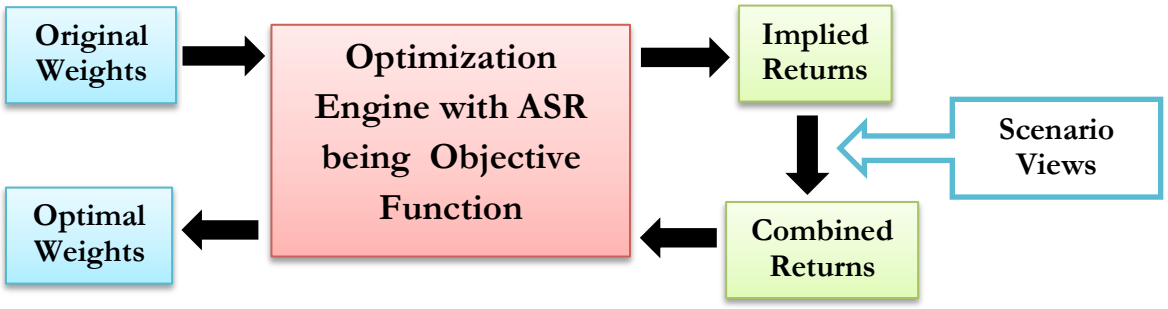
   As an example, the Alternative Sharp Ratio (ASR) represents a ratio of upside potentials against downside tail risk, and is defined as:

$$\text{ASR} = \frac{\sum_i e_i w_i}{z^- \sigma} + \frac{1}{2}\frac{\sum_i w_i (z_i^+ \sigma_i)^2}{z^- \sigma} - \frac{1}{2} z^- \sigma,$$

where:

$e_i$ is the excess return of asset *i*,
$w_i$ is its percentage weight in the portfolio,
$\sigma_i$ is the volatility of asset *i*,
$\sigma$ is portfolio volatility,
$z^+ = \frac{\max(z_{cf}(z_c^+),0)}{z_c^+}$, with
$z_{cf}$ being the Cornish-Fisher expansion factor to measure the "tail risk" of the portfolio for probability 1-$\alpha$
$z_c^+$ being the critical value for probability $\alpha$ (eg, 2.33 at 1%) and
$z^- = \frac{\min(z_{cf}(z_c^\pm),0)}{z_c^-}$ with
$z_c^-$ being the critical value for probability 1-$\alpha$ (eg, -2.33 at 99%).

**Computation Process:**




The computation process first computes a set of implied returns based on a set of weights. Implied returns are always defined with respect to a specific objective function. These implied returns can then be combined with scenario views, from which we can obtain the new optimal weights by re-optimizing the portfolio using the combined returns.

By taking the partial derivative of ASR with respect to the weight of asset i and rearranging the terms, we can obtain a rough approximation for the implied return of the $i^{th}$ investment, as follows:

$$e_i^* = (\text{ASR} + z_\pi^- \sigma)\frac{\partial z^- \sigma}{\partial w_i} - \frac{1}{2}(z_i^+ \sigma_i)^2 = \text{ASR}\frac{\partial z^- \sigma}{\partial w_i} + z^- \sigma \frac{\partial z^- \sigma}{\partial w_i} - \frac{1}{2}(z_i^+ \sigma_i)^2$$

This formula suggests that the "upside" volatility of an individual investment lowers its implied return, while an investment's contribution to the portfolio's "downside" fat-tail increases its implied return, which is consistent with our intuition. Given that ASR is a fourth-order function, the more precise optimization and reversed optimization will need to be computed using numerical methods.

The ASR optimization engine on the HedgeSPA platform, when combined with the Black-Litterman view generator, allows our users to derive optimal asset allocations by incorporating scenario views in a pragmatic and user-friendly manner.

*Disclosure:*

*This white paper describes a new methodology that has almost completed its implementation (if not already in testing) on our platform. As part of our planned release process, we are publishing this white paper as one of the final steps to collect feedback from prospective users prior to releasing the actual software functionalities on our platform. The explicit purpose of this process is to collect and incorporate user feedback into features that are about to be formally released; therefore, the final release may appear different from what is described in this white paper solely at the discretion of HedgeSPA. Moreover, such a process may result in HedgeSPA publishing modified versions of this white paper prior to releasing the actual software functionalities on our platform.*

*HedgeSPA users can derive quantitatively rigorous recommendations using our advanced analytics without manually scraping data from multiple sources and doing massive complicated computations. While the user may ultimately decide to come up with an alternative macroeconomic scenario or put on a different hedge, he can easily and quickly redo these highly complex, actionable calculations in a matter of minutes instead of waiting hours if not overnight for traditional solutions to complete similar calculations yet having much less accurate results while markets move. That is how HedgeSPA's solid investment analytics solution can save our users form common pitfalls that ruin countless other portfolios.*





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