Case Study: More Robust Methods to Estimate Capital Adequacy

Overview:

Asset manager Brian is managing a life insurance company's portfolio valued at $500 million. The portfolio is quite similar to AIG's investment portfolio published as of August 2014 (See **Figure 1**). Even though the model portfolio below is represented by investment funds instead of underlying assets, it provides a sufficiently realistic representation of portfolio characteristics. In order to cover potential, unexpected losses at a given confidence level and timeframe, an insurance company is required to set aside certain economic capital (as defined by its regulators) to ensure that it has sufficient assets to meet its obligations regardless of what happens in the markets.

Buy | Sell | Fields: Default | Calc. Horizon: 1 Year | Use Active Return | Exclude Cash | Use Weekly | Import | Export

Description			Position	Weight	Market	Risk	Return	
Product	Asset Class	Currency	Actual	Actual	Last Price	Volatility	Historical	6m Beta
Cash in USD	Cash	USD	19,999,999.01	4.00%	1.00	0.00%	0.00%	0
MSCI World Index ETF (iShares)	Equity	USD	15,000,000.00	3.00%	73.37	10.98%	14.20%	0.94
Morgan Stanley Instl Corporate Bond P	Fixed Inco...	USD	25,000,000.00	5.00%	11.07	3.27%	7.65%	-0.83
Optimum Fixed Income A	Fixed Inco...	USD	33,000,000.00	6.60%	9.62	2.60%	3.27%	-0.54
Pimco Municipal Income Fund III	Fixed Inco...	USD	90,000,000.50	18.00%	10.70	5.58%	14.16%	-0.48
Vanguard Mortgage-Backed Securities Index Fund	Fixed Inco...	USD	85,000,000.00	17.00%	28.36	2.40%	3.63%	-0.63
iShares Dow Jones US Real Estate (ETF)	Real Estate	USD	40,000,000.00	8.00%	70.91	10.86%	13.42%	-0.19
iShares Emerging Markets High Yield Bond ETF	Equity	USD	28,000,000.00	5.60%	50.78	6.97%	9.22%	-0.03
iShares International Treasury Bond ETF	Fixed Inco...	USD	70,000,000.00	14.00%	100.54	5.08%	7.13%	-0.73
iShares U.S. Treasury Bond ETF	Fixed Inco...	USD	10,000,000.48	2.00%	24.68	2.60%	1.69%	-0.52
iShares iBoxx $ High Yield Corporate Bd	Fixed Inco...	USD	12,000,000.00	2.40%	93.02	3.22%	5.24%	-0.07
iShares iBoxx $ Invest Grade Corp Bond	Fixed Inco...	USD	72,000,000.00	14.40%	117.53	4.34%	6.82%	-1.01
Total			500,000,000.00	100.00%			7.92%	-0.52

Figure 1: Investment portfolio of the Insurance Company

Business Problem:

Like most insurance companies' asset managers, Brian would like to minimize his company's capital adequacy requirements. Current market practice is that when regulators have reasonable doubts over the accuracy of any capital adequacy calculations, they may impose a significant penalty factor to cover any expected margin of error, which will translate into additional non-deployable reserves. The specific implementation of capital adequacy rules is dependent on jurisdictions, but it is typically based on 2 pillars:

Pillar I – VaR at 99.97% confidence.

Pillar I covers risk-based capital requirements for statistically predicable risks such as credit risk, market risk, and operational risk. The (most commonly used) 99.97% confidence interval will yield events so rare that they might happen in only 1 year out of every 3,300. The goal is for the insurance company to have sufficient capital to pay claims even if a very bad market event happens.

Pillar II – Scenarios – Disease, Military Conflicts, etc.

A series of stress tests are then applied to determine capital requirements complementary to Pillar I requirements, basically to capture possible events which data are not reflected by market statistics. Pillar II provides a framework for dealing with systematic risk, pension risk, concentration risk, strategic risk, reputational risk, liquidity risk, and legal risk.

Computing Pillar I requirement

Portfolio	Historical Return	Volatility	VaR	MaxDD	Beta	SR	ASR	Skewness	Kurtosis	Treynor Ratio	Jensen's Measure	Semi-Deviation
Actual Portfolio	7.91%	3.15%	13.25%	2.11%	-0.52	1.8742	1.5513	-0.24	3.35	0.0582	0.0587	3.46%

Figure 2: Portfolio Statistics generated by HedgeSPA platform

One traditional way of calculating Pillar I is to scale the portfolio's annualized volatility to a standard-normal VaR at 99.97%. By using this traditional method, Brian multiplies his portfolio's volatility of 3.15% by the critical value at 99.97%, which is approximately 3.43, to obtain an estimate of 10.80% for VaR. On the other hand, using fourth order mathematics by incorporating skewness and kurtosis, his VaR will be 13.25%. 10.80% may appear more desirable at first glance, but Brian knows that it is an underestimated value, because a standard normal assumption at this level of extreme confidence is grossly unrealistic. Typically, regulators will demand a "penalty ratio" that normally falls between 1.3-1.5, leading to a VaR between 14.0% and 16.2%.

Penalty Ratio	1.3	1.4	1.5
Penalty-Adjusted VaR	14.04%	15.12%	16.20%
Pillar I Economic Capital ($ in million)	70.2	75.6	81

Figure 3: Penalty-Adjusted VaR of the Portfolio

Brian generates the table showing the penalty-adjusted VaR's and sees that even for the conservative penalty factor of 1.3, his VaR will exceed the estimate from a more accurate measure[1] of 13.25%. A penalty ratio of 1.3 will be unusually generous at such an extreme confidence level; in practice, the penalty is more likely to be closer to 1.5. By reporting a more accurate VaR, Brain believes that he is likely to end up with a more capital adequacy treatment for Pillar I. Although the result from a more accurate calculation seems more aggressive at first glance, it is far more likely to result in a lower capital charge than an inaccurate calculation – a less accurate calculation leads to more inaccuracies and larger penalties. Therefore, Brian expects the extremely bad 0.03% one-year loss to be 13.25% of the entire portfolio, so that the insurance company will need to prepare for Pillar I economic capital of about $66 million instead of about $81 million, reducing by the requirement by a meaningful amount of about $15 million.

[1] The system applies Cornish-Fisher expansion, which includes skewness and kurtosis; with the 1st to 4th order moments all taken into account, the method is much more accurate than the case where only the 1st and 2nd order moments are considered in the traditional method. (See Technical Appendix)

Computing Pillar II requirements.

Brian now turns to examine the shocks from market scenarios that would contribute additional losses to the portfolio. He has chosen two scenarios, the *U.S. Tech Bubble* and the *Obama's Foreign Policies Failure,* to represent market crises not captured by statistical data. Each event has the potential to affect the whole market, thus changing the performance of the portfolio.

Brian calculates Pillar II by running a regression[2] on the chosen market indices and portfolio daily returns. For each scenario, Brian chooses what market indices to use and the percentage changes resulted from any particular event. After running the regression, Brian obtains the scenario shocks on the portfolio as shown in the table below (**Figure 4**):

	US Tech Bubble Trouble			Obama's Foreign Policies Fail			
	S&P 500 Index	BofA Merrill Lynch US High Yield Master II Option-Adjusted Spread	intercept	S&P 500 Index	CBOE Interest Rate 10-Year T-Note	USDCNY	intercept
change %	-20%	40%		-5%	25%	19%	
coefficient	-0.41947	-0.18566103	-0.02099	0.186895	-0.216277907	0.360949	-0.02384
Calculated Position Change	-1.14%			-1.87%			

Figure 4: Portfolio Position Change Calculated Using Traditional Method

At first glance, the -1.14% shock from the *US Tech Bubble Trouble* scenario does not sound unreasonable, but the -1.87% shock under the *Obama's Foreign Policies Fail* scenario may seem fairly unrealistic, because Obama's foreign policies will involve some major global conflicts that are sure to have significant impact on the global economy and financial markets. With the results being so unconvincing, the regulators will most likely demand that the scenario shocks should be multiplied by a large factor or simply asked to use a minimum shock of say -10%, in the absence of any reasonably-sized scenario shocks.

Brian doubts the accuracy of using this method because turning individual asset returns into aggregated portfolio return will certainly "mute" the volatility on the aggregated time series. It will be more accurate if he uses asset-by-asset beta regression[3], a regression running on each asset that computes assets value changes. In this way, Brian can come up with an estimate by taking each asset into account – the alternative is to destroy information by aggregating portfolio returns and reducing the "degree of freedom" in the problem, and then project a shock by increasing the "degree of freedom" in the statistical problem. These steps are not helpful to maintaining statistical accuracy.

Now Brian can see why any platform that can perform asset-by-asset beta regression can lead to very different and far more credible portfolio shocks, or -3.02% and -7.12%, respectively (**Figure 5**).

Description		Position	Weight	Return	US Tech–Bubble Trouble			Obama's Foreign Policies Fail		
Product	Asset Class	Actual	Actual	Historical...	Pos Chg	Asst Chg	Scen Prc	Pos Chg	Asst Chg	Scen Prc
MSCI World Index ETF (iShares)	Equity	15,000,000.00	3.13%	19.16%	-0.69%	-22.08%	55.11	-0.86%	-27.62%	51.19
Morgan Stanley Instl Corporate Bond P	Fixed Income	25,000,000.00	5.21%	8.77%	-0.15%	-2.90%	10.80	-0.26%	-5.03%	10.56
Optimum Fixed Income A	Fixed Income	33,000,000.00	6.87%	3.16%	-0.06%	-0.93%	9.57	-0.13%	-1.84%	9.48
Pimco Municipal Income Fund III	Fixed Income	90,000,000.00	18.75%	15.59%	-0.24%	-1.26%	10.67	-0.51%	-2.70%	10.52
Vanguard Mortgage-Backed Securities Index Fund	Fixed Income	85,000,000.00	17.71%	3.42%	-0.17%	-0.98%	28.27	-0.77%	-4.37%	27.30
iShares Dow Jones US Real Estate (ETF)	Real Estate	40,000,000.00	8.33%	17.45%	-1.27%	-15.29%	58.96	0.24%	2.86%	71.59
iShares Emerging Markets High Yield Bond ETF	Equity	28,000,000.00	5.83%	11.19%	-0.29%	-5.00%	47.97	-3.41%	-58.44%	20.98
iShares International Treasury Bond ETF	Fixed Income	70,000,000.00	14.58%	6.06%	0.24%	1.64%	100.14	-0.99%	-6.79%	91.83
iShares U.S. Treasury Bond ETF	Fixed Income	10,000,000.00	2.08%	1.73%	-0.13%	-6.44%	23.33	-0.05%	-2.52%	24.31
iShares iBoxx $ High Yield Corporate Bd	Fixed Income	12,000,000.00	2.50%	5.73%	-0.19%	-7.57%	85.17	-0.20%	-8.00%	84.77
iShares iBoxx $ Invest Grade Corp Bond	Fixed Income	72,000,000.01	15.00%	7.25%	-0.05%	-0.36%	118.50	-0.17%	-1.16%	117.55
Total		480,000,000.00	100.00%	9.06%	-3.02%			-7.12%		

Figure 5: Portfolio Position Changes under HedgeSPA Platform

Although the scenario shocks do differ a lot at first glance, the asset-by-assets beta regression results seem more credible knowing the potentially large impact of these events. Furthermore, Brian can precisely see the position-level shocks of each asset, and he specifically identifies *iShares Emerging Markets High Yield Bond ETF* to be the "greatest contributor", which is yielding a portfolio loss. That gives Brian a chance to consider another investment product that may be less sensitive to this scenario. We shall make the simplifying assumption that in this case Pillar II economic capital is calculated based on the worst shock, which is the *Obama's Foreign Policies Fail* scenario, and then simply added to Pillar I requirement. Under such an assumption, Brian will prepare additional Pillar II economic capital of $36 million.

Conclusion

In order to better meet his company's capital adequacy requirements, Brian can see that using more systematic approaches may prove advantageous. The traditional ways to calculate extreme losses are known to be inaccurate due to well-known statistical issues, so that regulators are likely to impose "penalty factors" that may lead to overly conservative estimates of reserves. Brian can also see how new tools can provide more accurate calculations while reducing the manual efforts involved: running asset-by-asset beta regressions for a "small" 100-asset portfolio on spreadsheets can take several days, while similar calculations can be finished in seconds on the HedgeSPA platform. Naturally, Brian prefers a more natural and efficient way to get the job done, and prepares a total economic capital of $102 million for his portfolio (under our assumption of simple aggregation), accounting for roughly 20% of the entire portfolio. Please note that most aggregation rules are more complex in real-life jurisdictions, but this case serves to illustrate the improved methodology that can be supported by the HedgeSPA platform.

Technical Appendix

1. VaR: A portfolio's Value-at-Risk (VaR) is defined as the maximum *loss* in the portfolio value over a period of time, at a given level of confidence. In this case study, we specify the confidence interval to be 99.97%. Please note that, while we are using VaR for the purpose of a simplified illustration, some jurisdictions may prefer to use expected shortfall instead.

Traditional VaR

$$VaR_{\pi,\alpha} := -F^{-1}(\alpha),$$

where $F(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma_\pi} e^{-\frac{1}{2\sigma_\pi^2}(x-\mu_\pi)^2} \, dx$

Cornish-Fisher *VaR*

$$VaR_{\pi,\alpha} = -(\mu_{\pi,A} + Z_{cf\pi} * \sigma_{\pi,A})$$
$$= -\mu_{\pi,A} - Z_{cf\pi} * \sigma_{\pi,A}$$

where $Z_{cf\pi}(Z_\alpha) = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S_\pi + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)(K_\pi - 3) - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha){S_\pi}^2$.

- $Z_\pi{}^+ = \frac{\max(z_{cf\pi}(z_\alpha^+),0)}{z_\alpha^+}$, $Z_\pi{}^- = \frac{\min(z_{cf\pi}(z_\alpha^-),0)}{z_\alpha^-}$
- Cornish-Fisher Expansion:

$$Z_{cf\pi}(Z_\alpha) = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S_\pi + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)(K_\pi - 3) - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha){S_\pi}^2.$$

S: Skewness K: Excess Kurtosis= Plain Kurtosis-3

2. Portfolio beta regression and asset-by-asset beta regression

In this case study, the portfolio beta regression and asset-by-asset beta regression are used to identify the statistical relationship between certain key market indices and portfolio returns.

Procedure of running **portfolio beta regression**:

a) Obtain the time series of each asset prices and generate portfolio return time series based on asset weights;
b) Determine the scenario and its corresponding market indices. Run a regression on portfolio returns and the market indices, obtaining the following linear relationship:
$$Y = \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n,$$
where X_i is the change of each market index, β_i is the coefficient of the corresponding i-th index, and Y is the change of the portfolio;
c) Determine the change of each market index and obtain the portfolio-level shock using the formula in b).

Procedure of running **asset-by-asset beta regression**:

a) Obtain the time series of each asset prices and determine the scenario and its corresponding market indices.

b) Instead of computing portfolio returns, run a regression on asset returns and the market indices, which results in the following linear relationship:

$$Z_j = \beta_{j1}X_1 + \beta_2X_2 + \ldots + \beta_{jn}X_n,$$

where X_i is the change of each index, β_{ji} is the coefficient of the corresponding index i and j-th asset, and Z_j is the asset-level change of asset j;

c) Obtain portfolio position change by summing the weighted asset position change:

$$Y = W_1Z_1 + W_2Z_2 + \ldots + W_mZ_m,$$

where Wj is the asset weight, Zj is the position change of asset j, and Y is the portfolio-level shock of the portfolio.

HedgeSPA users can derive quantitatively rigorous recommendations using our advanced analytics without manually scraping data from multiple sources and doing massive complicated computations. While the user may ultimately decide to come up with an alternative macroeconomic scenario or put on a different hedge, he can easily and quickly redo these highly complex, actionable calculations in a matter of minutes instead of waiting hours if not overnight for traditional solutions to complete similar calculations yet having much less accurate results while markets move. That is how HedgeSPA's solid investment analytics solution can save our users form common pitfalls that ruin countless other portfolios.

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